

CAMBRIDGE A LEVEL · 9709

A Level Maths Formula Sheet

The MF19 list for pure, mechanics and statistics, plus what is not on it.

1 MF19 Pure: mensuration and algebra

Supplied in every 9709 paper.

Volume of a sphere	$V = \frac{4}{3}\pi r^3$
Surface area of a sphere	$A = 4\pi r^2$
Volume of a cone or pyramid	$V = \frac{1}{3} \times \text{base area} \times \text{height}$
Curved surface area of a cone	$A = \pi r \times \text{slant height}$
Arc length (θ in radians)	$s = r\theta$
Area of a sector (θ in radians)	$A = \frac{1}{2}r^2\theta$
Quadratic formula, for $ax^2 + bx + c = 0$ ($a \neq 0$)	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic series: nth term	$u_n = a + (n - 1)d$
Arithmetic series: sum of n terms	$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n - 1)d\}$
Geometric series: nth term	$u_n = ar^{n-1}$
Geometric series: sum of n terms ($r \neq 1$)	$S_n = \frac{a(1 - r^n)}{1 - r}$
Geometric series: sum to infinity ($ r < 1$)	$S_\infty = \frac{a}{1 - r}$
Binomial expansion (n a positive integer)	$(a + b)^n = a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \dots + b^n$
Binomial coefficient	${}^nC_r = \frac{n!}{r!(n - r)!}$
Binomial series (n rational, $ x < 1$)	$(1 + x)^n = 1 + nx + \frac{n(n - 1)}{2!}x^2 + \frac{n(n - 1)(n - 2)}{3!}x^3 + \dots$

2 MF19 Pure: trigonometry

Identities, compound angles and double angles.

Tangent identity	$\tan \theta = \frac{\sin \theta}{\cos \theta}$
Pythagorean identity	$\cos^2 \theta + \sin^2 \theta = 1$
Identity with sec	$1 + \tan^2 \theta = \sec^2 \theta$
Identity with cosec	$\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$
Compound angle: sine	$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
Compound angle: cosine	$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
Compound angles: tangent	$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
Double angle: sine	$\sin 2A = 2 \sin A \cos A$
Double angle: cosine	$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$
Double angle: tangent	$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

3 MF19 Pure: calculus and vectors

Standard derivatives and integrals; arbitrary constants are omitted.

Derivatives of x^n , $\ln x$ and e^x	$x^n \rightarrow nx^{n-1}, \ln x \rightarrow \frac{1}{x}, e^x \rightarrow e^x$
Derivatives of $\sin x$, $\cos x$ and $\tan x$	$\sin x \rightarrow \cos x, \cos x \rightarrow -\sin x, \tan x \rightarrow \sec^2 x$
Derivatives of $\sec x$ and $\operatorname{cosec} x$	$\sec x \rightarrow \sec x \tan x, \operatorname{cosec} x \rightarrow -\operatorname{cosec} x \cot x$

Derivatives of $\cot x$ and $\tan^{-1}x$	$\cot x \rightarrow -\operatorname{cosec}^2 x$, $\tan^{-1}x \rightarrow \frac{1}{1+x^2}$
Product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
Quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
Parametric differentiation	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$
Integrals of x^n ($n \neq -1$), $1/x$ and e^x	$x^n \rightarrow \frac{x^{n+1}}{n+1}$, $\frac{1}{x} \rightarrow \ln x $, $e^x \rightarrow e^x$
Integrals of $\sin x$, $\cos x$ and $\sec^2 x$	$\sin x \rightarrow -\cos x$, $\cos x \rightarrow \sin x$, $\sec^2 x \rightarrow \tan x$
Integral of $1/(x^2 + a^2)$	$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$
Integration by parts	$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$
Integral of $f'(x)/f(x)$	$\int \frac{f'(x)}{f(x)} dx = \ln f(x) $
Scalar product	$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 = a b \cos \theta$

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MF19 Mechanics and Probability & Statistics

Constant acceleration, summary statistics and probability distributions.

Uniformly accelerated motion	$v = u + at$, $s = \frac{1}{2}(u + v)t$
Uniformly accelerated motion	$s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$
Mean (ungrouped data)	$\bar{x} = \frac{\sum x}{n}$
Standard deviation (ungrouped data)	$\sqrt{\left(\frac{\sum(x - \bar{x})^2}{n}\right)} = \sqrt{\left(\frac{\sum x^2}{n} - \bar{x}^2\right)}$
Discrete random variables	$E(X) = \sum xp$, $\operatorname{Var}(X) = \sum x^2 p - \{E(X)\}^2$
Binomial distribution $B(n, p)$	$p_r = {}^n C_r p^r (1 - p)^{n-r}$
Binomial mean and variance	$\mu = np$, $\sigma^2 = np(1 - p)$
Geometric distribution $\operatorname{Geo}(p)$	$p_r = p(1 - p)^{r-1}$, $\mu = \frac{1}{p}$
Poisson distribution $\operatorname{Po}(\lambda)$	$p_r = e^{-\lambda} \frac{\lambda^r}{r!}$, $\mu = \lambda$, $\sigma^2 = \lambda$
Central Limit Theorem	$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$
Unbiased estimate of variance	$s^2 = \frac{\sum(x - \bar{x})^2}{n - 1}$

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Not on MF19: formulas to know

The syllabus expects you to use these, but MF19 does not list them.

Weight	$W = mg$
Newton's second law	$F = ma$
Momentum	momentum = mv
Kinetic and gravitational potential energy	$KE = \frac{1}{2}mv^2$, $PE = mgh$
Power (force in the direction of motion)	$P = Fv$
Laws of logarithms: product	$\log x + \log y = \log xy$
Laws of logarithms: quotient	$\log x - \log y = \log \frac{x}{y}$
Laws of logarithms: power	$k \log x = \log x^k$
Chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$